XXXII. *Compariſon between Sir* George Shuckburgh *and Colonel* Roy's *Rules for the Meaſurement of Heights with the Barometer*; *in a Letter to Col.* Roy, *F. R. S. from Sir* George Shuckburgh, *Bart. F. R. S.*

TO COLONEL ROY.

Welbeck Street,
April 20, 1778.

SIR,

Read May 7, 1778.

SINCE the printing of your ingenious memoir on the ſubject of meaſuring heights with the barometer, I have been naturally led to a compariſon of your rules and obſervations with my own[(a)]; and herein am not more pleaſed than ſurprized at the general correſpondency of our reſults, which carries with it the appearance of one and the ſame ſeries of experiments, rather than of diſtinct obſervations made with different inſtruments, in different countries, and by different perſons. That the ſtandard temperature or zero on the ſcale of the thermometer ſhould be found by each of us to fall in the ſame point to within one-third of a degree is, I think, truly ſurprizing; and I doubt not will evince to Mr. DE LUC the ſtrong probability there is of

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(a) Vide Phil. Tranſ. vol. LXVII.

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the neceſſity of correcting his rules. But although in this eſſential and fundamental part of the inquiry we agree, there are, nevertheleſs, ſome little circumſtances wherein we differ; it is the ſubject of this letter, SIR, to point out to you the degree of our differences, a compariſon that I had the pleaſure ſlightly to exhibit to you a few days ago, and which I truſt will not be found unimportant to thoſe who may be engaged in theſe purſuits: if, therefore, you judge theſe remarks of ſufficient moment, I will beg the favour of you to lay them before the Royal Society, as the beſt means of communicating them to the public.

The two chief cauſes of our difference are, the expanſion of quickſilver and the expanſion of air. I ſhall begin with the equation for quickſilver.

The mean temperature of ordinary barometrical obſervations, I apprehend, will generally be found to lie between 40° and 70° on FAHRENHEIT'S thermometer; now the mean expanſion in this range, according to your obſervation, is ,0323 inch on a column of 30 inches for 10° of heat; by my table it is only ,0304 inch, the difference ,0019 inch is equal to about 20 inches in the reſult of the height, when the temperature of the two barometers differs 10°, and this may reaſonably be expected only in a height of 3000 or 4000 feet. In an obſer-

obſervation on Mount Ætna, one of the greateſt acceſſible heights in Europe, the difference of temperature at the top and bottom might amount to 30°, and this would occaſion a difference of about five feet, which, I apprehend, may be reckoned inconſiderable in a height of 11,000 feet. In fact, in an obſervation on this mountain by Mr. DES-SAUSSURE it amounted to only 3½ feet. I may add, that your equation makes the computed height leſs than mine.

I proceed to the expanſion of air. Your equation is various according to the circumſtances, the difference therefore of our reſults will, according to the circumſtances, be various. The following table will give the quantity of this difference, *viz.* it ſhews how much your reſult is + or − mine upon one thouſand feet, according to different preſſures of the atmoſphere and different temperatures. The firſt column to the left hand contains the mean heat of the column of air between the two barometers; the figures in the horizontal line at top are the mean height of the two barometers, or mean preſſure of the atmoſphere; the common point of meeting in the different columns gives the difference of our reſult in feet, according to the reſpective circumſtances.

Mean heat.	Mean height of the two barometers in inches.										
°	30	29	28	27	26	25	24	23	22	21	20
32	0	0	0	0	0	0	0	0	0	0	0
42	+1	0	−1	−2	−3	−4	−5	−6	−7	−8	−9
52	+2	0	−2	−4	−6	−8	−10	−12	−14	−16	−17
62	+5	+2	−1	−4	−7	−10	−13	−16	−19	−22	−25
72	+8	+4	0	−4	−10	−12	−17	−21	−25	−29	−33

Thus, if the mean height of the barometer were 27 inches, and the temperature 52°, the difference of the reſults would be four feet in a thouſand; how far, therefore, this is of moment is left to the judgement of the obſerver. I conclude laſtly, SIR, with a compariſon of your actual obſervations made in Great Britain computed after my tables, deduced from a ſeries of obſervations made in Savoy. I have indeed only collected ſixteen of your obſervations; but as I have choſen ſuch as preſented themſelves as moſt proper, either on account of their heights or temperatures, I imagine theſe will be very ſatisfactory.

Place of obſervation.	True height in feet geometrically.	By my tables,			By Col. ROY's tables,			Mean temp. of the obſ.
		Computed height.	Error in feet.	Error on 1000 feet.	Computed height.	Error in feet.	Error on 1000 feet.	
Station at Weem and top of Bolfrack's Cairn,	1076.5	1074.2	− 2.3	− 2.1	1075.5	− 1.0	− 0.9	° $58\frac{1}{2}$
Ditto ſtation and top of Dull-craig,	1244.5	1240.6	− 3.9	− 3.1	1244.4	− 0.1	− 0.0	56
Ditto ſtation and South obſ. on Schehallien,	2098.0	2096.0	− 2.0	− 0.9	2091.3	− 6.7	− 3.2	$52\frac{1}{2}$
Ditto ſtation and Weſt ſummit of Schehallien,	3281.0	3292.5	+ 11.5	+ 3.5	3279.0	− 2	− 0.6	$50\frac{1}{2}$
	——	3261.3	− 19.7	− 6.1	3252.5	− 29.5	− 8.9	$46\frac{1}{8}$
Carmichael-well and top of Tinto,	1642.5	1653.6	+ 11.1	+ 6.7	1646.9	+ 4.4	+ 2.7	$54\frac{1}{2}$
	——	1645.0	+ 3.5	+ 2.1	1642.7	+ 0.2	+ 0.1	48
Level of Hawkhill and ſmall rock on Arthur's Seat,	702.4	704.2	+ 1.8	+ 2.5	703.	+ 1.3	+ 1.9	$20\frac{1}{2}$
Baſe of Hawkhill Obſervatory and bottom of the ſmall rock,	684.0	684.1	+ 0.1	+ 0.1	686.7	+ 2.7	+ 3.9	17
	——	685.4	+ 1.4	+ 2.0	687.0	+ 3.0	+ 4.4	$68\frac{1}{2}$
Hawkhill garden-door and bottom of the rock on Arthur's Seat,	730.8	723.0	− 7.8	− 10.7	721.8	− 9.0	− 11.3	$29\frac{1}{2}$
	——	745.8	+ 15.0	+ 20.6	748.4	+ 17.6	+ 24.2	71
Linhouſe and Eaſt Cairn hill,	1176.6	1182.5	+ 5.9	+ 5.0	1180.0	+ 3.4	+ 2.9	$30\frac{1}{2}$
Carnarvon Quay and Moel Eilio,	2371.	2397.4	+ 26.4	+ 11.2	2393.3	+ 22.3	+ 9.4	$62\frac{3}{8}$
Carnarvon Quay and Peak of Snowdon,	3555.	3563.1	+ 8.1	+ 2.3	3551.3	− 3.7	− 1.0	$56\frac{1}{2}$
	——	3551.5	− 3.5	− 1.0	3548.4	− 6.6	− 1.9	53
		Mean error		+ 2.0	Mean error		+ 1 4	

Thus

Thus it ſeems, that the error of my tables, from a mean of all theſe obſervations, amounts to + $\frac{20}{10000}$; of yours, to + $\frac{14}{10000}$: but it muſt be remarked, that the ſtandard temperature, from whence I compute, is 31°,24 of FAHRENHEIT, whereas in your computations it is aſſumed at 32°,0; this difference of 0°,76 is equal to $\frac{18}{10000}$ in the correction for the expanſion of the air: if then we were to ſet out from the ſame zero, *viz.* 32° (which I have propoſed, ſee p. 569. of my memoir, Phil. Tranſ. vol. LXVII.) the error of my tables, according to your obſervations, would become only $\frac{2}{10000}$, that of yours remaining $\frac{14}{10000}$. I would by no means from hence conclude, that any preference is to be given to the former, but would ſay, that in moſt practical obſervations, in theſe countries at leaſt, it is indifferent which method is uſed. Theſe ſame compariſons alſo afford us another piece of information, *viz.* that under ſimilar conditions the denſity of the atmoſphere is the ſame, whether under the parallel of 46 or 56 degrees of latitude. Till, therefore, more accurate obſervations than thoſe of Mr. BOUGUER (a) can be obtained in the neighbourhood of the

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(a) Mr. BOUGUER's obſervations I hold inadequate in ſuch an inquiry, not only on account of their incorrectneſs (for they are related only to the ¼ of a French line), but particularly inconcluſive, allow them all the preciſion you pleaſe, as they were not ſynchronous; inſomuch, that we are uncertain whether thoſe obſervations, which ſhould be correſponding in point of time, were made within ſix months of each other: and with regard to the temperature of ſome of them the equator, I ſhould be extremely cautious how I admitted a latitudinal equation; nor do I think the ſingle obſervation, related in *The Voyage towards the North Pole*, of ſufficient authority itſelf to eſtabliſh ſuch a theory upon(b).

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them we are ſtill more at a loſs, having a range of no leſs than 38° to aſſume it in (vide Voyage au Perou, p. 29.). The mean, however, of his five obſervations, according to my computation, would bring the zero of the ſcale to about 40° of FAHRENHEIT: but till it can be proved, that in this uncertainty of 38° we have, in fixing upon the mean, got the true temperature to within 8°; and alſo, that during many months the barometer in the ſame place had never altered a ſingle line: till then, I ſay, no fair concluſion, in point of theory, can be drawn from theſe experiments, for with ſuch ſuppoſed errors or variations the Peruvian obſervations will agree exactly with thoſe in this climate.

(b) It ſeems extraordinary, that the heights of the quickſilver, obſerved on the ſea-ſhore with one of Mr. RAMSDEN's barometers, ſhould differ a quarter of an inch from the height of the ſame obſerved the ſame day, and almoſt at the ſame hour, with Mr. NAIRNE's marine barometer on board a ſhip (vide p. 135 and 148. of *The Voyage towards the North Pole*). This difference, therefore, I think remains to be explained, if the experiment is to be made uſe of in this inquiry; for this ſame marine barometer, compared a few days ago at Mr. IBBETSON's, ſecretary to the board of longitude, with one of my own, that I uſed in Savoy, agreed with it to within 0,04 inch. It may alſo be remarked, that Lord MULGRAVE's obſervation in lat. 79° 44′ brings the zero to about 64°; ſo that between the lat. 56° and 79° (equal 23°) the zero of the ſcale moves through a ſpace of no leſs than 32°; whereas, between the lat. 46° and 56° it is perfectly ſtationary, at leaſt as to ſenſe, having altered only one-third of a degree: which great want of proportion, I think, is of itſelf ſome argument againſt the exiſtence of ſuch a latitudinal equation. Moreover, Mr. BOUGUER's obſervations brings the zero of the ſcale on the ſame ſide of 32° with Lord MULGRAVE's, *viz.* to about 40°, ſo that, if any deduction is to be drawn from theſe obſervations, it is, that the air, both at the equator and at the pole is heavier than in middle latitudes. which is no very probable conjecture, and, I apprehend, more than is intended to be proved. I ſhould have aſked pardon for the freedom with which I have diſcuſſed this inquiry, were I not aſſured that the acquaintance and friendſhip of my Lord MULGRAVE, which I have experienced for ſome years, will lead him to attribute it to its proper motive.

I ſhall now beg leave to conclude with what I flatter myſelf will not here appear improper, a new rule for reducing the obſervations, and which I hope will be found particularly commodious, as it requires no logarithms, nor any other than the following ſhort table, which may be engraven upon the ſcale of a thermometer, and therefore, always accompanying the inſtrument, will ſerve for computing the obſervations upon the ſpot, (if the height ſhould not exceed 4000 or 5000 feet) which, I apprehend, will frequently be found very ſatiſfactory.

Ther.		
°	Feet.	The value of one-tenth of an inch of quickſilver on the barometer, expreſſed in feet in the atmoſphere when the barometer ſtands at 30 inches, according to the different temperatures.
32	85.86	
35	87.49	
40	88.54	
45	89.60	
50	90.66	
55	91.72	
60	92.77	
65	93.82	
70	94.88	
75	95.93	
80	96.99	

The adjoined table gives the value of $\frac{1}{10}$th of an inch on the barometer in feet in the atmoſphere, when the quickſilver ſtands at 30 inches, for every five degrees of temperature from 32° to 80°; and for any other height of the barometer it will be in the inverſe ratio of that height to 30 inches. Thus, let A be the mean height of the two barometers in inches; α the difference of the two barometers in 10ths of an inch; β the number taken out of the adjoined table; x the height in feet; we have then the following expreſſion, $\frac{30\alpha\beta}{A}=x$, the height required.

I have the honour to be, &c.